Matheus Andrade Ribeiro de Moura Horácio

PhD candidate in Mathematics • Differential Geometry and Geometric Analysis

Departamento de Matemática, Universidade de Brasília, 70910-900 Brasília, DF, Brazil

m.a.r.m.horacio@mat.unb.br • matheus.andrade5488@gmail.com • horatiomarm.com

ORCID 0009-0001-5658-7467 • Lattes lattes.cnpq.br/6896641753246997 • Papers horatiomarm.com/en/research.html

RESEARCH PROFILE

I am a PhD candidate in differential geometry at the Universidade de Brasília, and my main mathematical interests are closely related to the Ricci flow. More precisely, I investigate rigidity and classification problems for gradient Ricci solitons and for other soliton-type equations. In my doctoral thesis, I study gradient Ricci solitons and almost Ricci solitons with harmonic Weyl curvature, Kähler–Ricci solitons, and self-shrinkers of the mean curvature flow. Recently, I have worked on the conjecture of Huai-Dong Cao on shrinking Ricci solitons with constant scalar curvature, and on its counterpart for quasi-Einstein manifolds, by means of the focal set of the potential function.

Interests Ricci and Kähler–Ricci solitons • quasi-Einstein manifolds • warped products • self-shrinkers.

EDUCATION

2023 – present **PhD in Mathematics (in progress)**, Universidade de Brasília (UnB), Brazil
Advisor: João Paulo dos Santos. Thesis: *Rigidity and local structure of gradient Ricci solitons, almost Ricci solitons and self-shrinkers*. Defense expected in February 2027.

2021 – 2023 **MSc in Mathematics**, Universidade de Brasília
Advisor: João Paulo dos Santos. Dissertation on curvature estimates for four-dimensional gradient Ricci solitons, defended on 6 March 2023. repositorio.unb.br/handle/10482/49663
Examiners: Tarcísio Castro Silva (UnB) and Ernani Ribeiro Jr. (UFC).

2017 – 2020 **BSc in Mathematics**, Universidade de Brasília

RESEARCH PAPERS

Peer-reviewed article

[1] Local structure of gradient almost Ricci solitons with harmonic Weyl tensor
V. Borges, **M. A. R. M. Horácio**, J. P. dos Santos
The Journal of Geometric Analysis **36** (2026), no. 8, Paper No. 256. Published online 6 July 2026.
doi:10.1007/s12220-026-02507-9 | arXiv:2510.13057

Single-author preprints

[2] Rigidity of compact quasi-Einstein manifolds with boundary and low-dimensional focal sets
arXiv:2611.16314 | November 2026 | 27 pages

[3] Rigidity of shrinking Ricci solitons with low-dimensional focal sets
arXiv:2611.02740 | November 2026 | 28 pages

[4] On Ricci solitons whose level hypersurfaces have parallel second fundamental form
arXiv:2610.11209 | October 2026 | 14 pages

[5] Rigidity of nonsteady gradient Kähler–Ricci solitons with constant scalar curvature
arXiv:2609.29359 | September 2026 | 5 pages | *results contained, in strengthened form, in* [7]

[6] Multiply warped products with harmonic Weyl tensor and zero radial Weyl curvature
arXiv:2609.09759 | September 2026 | 14 pages

Joint preprints

[7] Rigidity of Kähler–Ricci solitons with constant scalar curvature
M. Horácio, W. Li, J. Ou, G. Wu, D. Zhou
arXiv:2610.11935 | October 2026 | 18 pages | submitted to *Calc. Var. Partial Differential Equations*

[8] The Ricci tensor of a gradient Ricci soliton with harmonic Weyl tensor
V. Borges, **M. A. R. M. Horácio**, J. P. dos Santos
arXiv:2510.11939 | October 2025, revised October 2026 | 17 pages

In preparation

[9] Gradient almost Ricci solitons with harmonic Weyl tensor and exactly three Ricci eigenvalues
V. Borges, **M. A. R. M. Horácio**, J. P. dos Santos

[10] On the rigidity of complete self-shrinkers with constant mean curvature length
M. A. R. M. Horácio, J. P. dos Santos

SELECTED RESULTS

[3], [2] Shrinking Ricci solitons with constant scalar curvature and their focal sets
Huai-Dong Cao conjectured that every complete gradient shrinking Ricci soliton with constant scalar curvature is rigid, that is, a quotient of the product of an Einstein manifold and a Euclidean space. On such a soliton, $R = k\lambda$ for some integer k, and the minimum set of the potential function is a compact minimal submanifold of dimension k, called its focal set. The main result of [3] states that, in every dimension, the focal set is totally geodesic whenever $k = 2$ or $k = 3$. Combined with a theorem of Li, Li, Ou, Qu, Wu and Zhou, it implies that the conjecture holds when $R \leq 3\lambda$. An independent argument in [3] also establishes the conjecture in dimension five. In [2], it is proved that a compact m-quasi-Einstein manifold with boundary, $m > 1$ and constant scalar curvature is rigid whenever the critical set of its potential function has dimension two or three. In particular, it is rigid in dimension five.

[5], [7] Kähler–Ricci solitons with constant scalar curvature
In [5], it is proved that a complete shrinking or expanding gradient Ricci soliton with constant scalar curvature is rigid if and only if $\mathscr{L}_{\nabla f}\mathrm{Ric} = 0$. As a consequence, every complete shrinking or expanding gradient Kähler–Ricci soliton with constant scalar curvature is rigid, and the conjecture of Huai-Dong Cao holds for Kähler–Ricci shrinkers in every dimension. In [7], in collaboration with W. Li, J. Ou, G. Wu and D. Zhou, the criterion is extended: if $\mathscr{L}_{\nabla f}\mathrm{Ric}$ is nonnegative or nonpositive everywhere, then the soliton is rigid, with no assumption on the scalar curvature.

[1], [8], [6], [4] Harmonic Weyl curvature and multiply warped products
In [1], in collaboration with V. Borges and J. P. dos Santos, we prove that the Ricci tensor of a gradient almost Ricci soliton with harmonic Weyl tensor has at most three distinct eigenvalues, and that every nontrivial example is locally a multiply warped product with a one-dimensional base and at most two Einstein fibers, and we classify those with exactly two distinct eigenvalues. These results extend a result by Catino, who assumes, in addition, that the Weyl tensor is radially flat, and a result by Kim, who considers the four-dimensional case. The multiply warped product structure of gradient Ricci solitons is further studied in [8], [6] and [4].

RESEARCH COLLABORATIONS

2023 – present **Solitons with harmonic Weyl curvature.** With Valter Borges (Universidade Federal do Pará) and João Paulo dos Santos (UnB). Papers [1] and [8].

2026 – present **Kähler–Ricci solitons with constant scalar curvature.** With Wenqi Li (East China Normal University), Jianyu Ou (Xiamen University), Guoqiang Wu (Zhejiang Sci-Tech University) and Detang Zhou (Universidade Federal Fluminense). Paper [7].

TALKS

Geometry Seminar, Department of Mathematics, Universidade de Brasília

21 Oct 2026 *Structure and rigidity of gradient Ricci solitons.* horatiomarm.com/seminario.pdf

3 Jun 2026 *Local structure of almost Ricci solitons with harmonic Weyl curvature* (in Portuguese). horatiomarm.com/meus_textos/seminario0306.pdf

27 May 2026 *Topological classifications revealed by the Ricci flow* (expository, in Portuguese; also given on 3 August 2022). horatiomarm.com/meus_textos/seminario2705.pdf

Undergraduate presentations

2019 *An introduction to the Ricci flow and Einstein manifolds* (with J. P. dos Santos). XI ENAPETMAT, national meeting of the PET-Matemática groups, Brasília. horatiomarm.com/meus_textos/TrabalhoENAPETMAT.pdf

2018 – 2019 Five talks in the PET-Matemática seminar, UnB, on geometry and geometric flows.

TEACHING AND OUTREACH

2019 – 2021 Teaching assistant, UnB: Calculus 2 (2019, volunteer), Differential Geometry 1 (2020, volunteer) and Analysis 3 (2021).

2020 Short course *A brief introduction to differentiable manifolds*, PET-Matemática, UnB. Lecture notes: horatiomarm.com/meus_textos/MinicursoPETMAT.pdf

2019 Short course *Introduction to LaTeX* (with M. S. Luiz and G. D. Couto), extension course, UnB.

2019 Extension project *Differential and integral calculus for gifted secondary-school students*, coordinated by Luciana M. D. de Ávila Rodrigues (UnB).

SCHOLARSHIPS

2023 – present Doctoral scholarship, CAPES (Brazilian federal agency for graduate education).

2021 – 2023 Master's scholarship, CNPq (Brazilian National Research Council).

2018 – 2020 Undergraduate fellowship, Programa de Educação Tutorial (PET-Matemática), UnB.

FURTHER INFORMATION

Writings Lecture notes and expository texts: horatiomarm.com/en/writings.html

Forums Mathematics Stack Exchange: math.stackexchange.com/users/508844
MathOverflow: mathoverflow.net/users/119418

Languages Portuguese (native), English (fluent).